

Calculus A Final Review Worksheet #1

Name:

Key

$$1. \lim_{x \rightarrow -3} (-2+x)^3 = (-2-3)^3$$

$$= \boxed{-125}$$

$$2. \lim_{x \rightarrow -2} \frac{x^2+x-2}{2+x} = \lim_{x \rightarrow -2} \frac{(x+2)(x-1)}{(x+2)}$$

$$= -2-1$$

$$= \boxed{-3}$$

3. Using the definition of derivative find the derivative of $y = x^2 - 2x$.

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \rightarrow \lim_{\Delta x \rightarrow 0} \frac{(x+\Delta x)^2 - 2(x+\Delta x) - (x^2 - 2x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} [x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x - x^2 + 2x]$$

$$= \lim_{\Delta x \rightarrow 0} \frac{1}{\Delta x} [\Delta x(2x + \Delta x - 2)]$$

$$= \boxed{2x - 2}$$

4. Find $\frac{dy}{dx}$ for $y = \sqrt[3]{x^2}(x+1) = x^{2/3}(x+1)$

$$\frac{dy}{dx} = x^{2/3}(1) + \frac{2}{3}x^{-1/3}(x+1)$$

$$= x^{2/3} \cdot \frac{3x^{1/3}}{3x^{1/3}} + \frac{2x+2}{3x^{1/3}}$$

$$= \boxed{\frac{5x+2}{3\sqrt[3]{x}}}$$

product rule

or → distribute first: $y = x^{5/3} + x^{2/3}$

$$\frac{dy}{dx} = \frac{5}{3}x^{2/3} + \frac{2}{3}x^{-1/3}$$

then factor

$$= \frac{x^{-1/3}}{3} (5x + 2)$$

$$= \boxed{\frac{5x+2}{3\sqrt[3]{x}}}$$

5. Find $\frac{d^2y}{dx^2}$ for $y = \frac{x+4}{x-1}$

$$\frac{dy}{dx} = \frac{(x-1)(1) - (x+4)(1)}{(x-1)^2}$$

$$= -\frac{5}{(x-1)^2}$$

$$= -5(x-1)^{-2}$$

$$\frac{d^2y}{dx^2} = 10(x-1)^{-3}(1)$$

$$= \boxed{\frac{10}{(x-1)^3}}$$

6. Find y' for $2y^2 - 2xy = x^3$

$$4y \frac{dy}{dx} - 2x \frac{dy}{dx} - 2y = 3x^2$$

$$\frac{dy}{dx}(4y - 2x) = 3x^2 + 2y$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2 + 2y}{4y - 2x}}$$

7. Find the equation for the tangent line of $y = 2x^2 + x - 2$ at the point where $x = 2 \rightarrow y = 2(4) + 2 - 2$
 $y = 8$

$$y' = 4x + 1$$

slope @ $x = 2$

$$m = 4(2) + 1 = 9$$

$$y - 8 = 9(x - 2)$$

8. Suppose the position equation for a moving object is given by $s(t) = -16t^2 - 4t + 1$. Find the velocity when $t = 1$

$$v(t) = s'(t) = -32t - 4$$

$$v(1) = -32 - 4$$

$$v(1) = -36$$

9. Find the absolute maximum and absolute minimum of f on the interval $(0, 2)$, $f(x) = \frac{x^3 - x^2 - 2x}{x}$

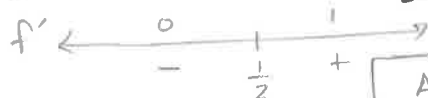
first simplify:

$$f(x) = x^2 - x - 2$$

$$f'(x) = 2x - 1$$

$$0 = 2x - 1$$

critical # @ $x = \frac{1}{2}$



Absolute Minimum
@ $(\frac{1}{2}, -\frac{9}{4})$

open interval, so no end points to check.

$$f(\frac{1}{2}) = 2(\frac{1}{8} - \frac{1}{4} - 1) = -\frac{9}{4}$$

10. The radius of a circle is measured at 4 inches. If the measurement is correct to within .02 inch, use differentials to estimate the propagated error in the area of the circle.

$$A = \pi r^2$$

$$\frac{dA}{dr} = 2\pi r$$

$$dA = 2\pi r dr$$

$$dA = 2\pi(4)(\pm 0.02)$$

$$dA = \pm 0.16\pi$$

$$\approx \pm 0.503$$

$$dr = (\pm 0.02)$$

11. Find $f'(x)$ for $f(x) = \frac{1}{2x - e^{2x}} \rightarrow f(x) = (2x - e^{2x})^{-1}$

$$f'(x) = -\frac{2 - 2e^{2x}}{(2x - e^{2x})^2}$$

$$f'(x) = \frac{2e^{2x} - 2}{(2x - e^{2x})^2}$$

12. Differentiate $f(x) = x^{3x^2}$

$$\ln y = 3x^2 \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = 3x^2 \frac{1}{x} + 6x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = 3x(1 + 2 \ln x)$$

$$\frac{dy}{dx} = 3x(1 + 2 \ln x) x^{3x^2}$$

$$\frac{dy}{dx} = 3x^{3x^2+1} (1 + 2 \ln x)$$

$$\text{LCD: } 5x(x^2 - 1)$$

$$y' = \frac{3}{x} + \frac{2x-1}{5x(x-1)} - \frac{4x}{x^2-1} \cdot \frac{5x}{5x}$$

$$y' = \frac{15(x^2-1) + (2x-1)(x+1) - 20x^2}{5x(x^2-1)}$$

$$y' = \frac{-3x^2 + x - 16}{5x(x^2-1)}$$

13. Take the derivative: $y = \ln \frac{x^3 \sqrt{x^2 - x}}{(x^2 - 1)^2}$

Expand:

$$y = 3 \ln x + \frac{1}{5} \ln(x^2 - x) - 2 \ln(x^2 - 1)$$